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A Stochastic Routing Model for Telecommunication Network Planning

by

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ABSTRACT

We present an off-line routing model that takes into account several planning features for circuit-switched networks. Applications of the model are surveyed. In particular, it is shown how the routing model can be incorporated into a procedure to evaluate network performance under reliability considerations.

Keywords: Routing, circuit-switching, network performance, reliability.

RÉSUMÉ

Nous présentons un modèle de planification d'acheminement d'appels pour des réseaux de commutation par circuit. Des applications du modèle sont montrées. Nous présentons comment le modèle peut être incorporé dans une procédure d'évaluation de la performance d'un réseau en tenant compte des considérations de fiabilité.

Mots-clés: acheminement d'appels, commutation par circuits, performance de réseaux, fiabilité.

1 Introduction

In recent years, two technologies have had an important impact in circuit-switched networks planning: the switches with stored program control and the introduction of fiber optics. The new switching technology has provided a new flexibility to the way calls are routed in the network. This flexibility did not exist in traditional hierarchical routing and has made it possible the idea of optimizing the routing with respect to some measure of performance. On the other hand, with the introduction of fiber optics, fewer links with huge capacity are being used to route a good part of the demand, thus making the reliability problem much more important. There is, therefore, the need for new types of mathematical models and computational tools to face the new problems created by the introduction of the latest technologies in the field.

Up to very recently, the only available routing technology for circuit-switched networks was hierarchical routing (see Wilkison (1956)). As switching techniques evolved toward more refined routing, telephone companies have been trying to set rules such that the call, when it arrives, is routed in a way that optimizes some performance or cost measure. This idea have given birth to some clever adaptive routing techniques based on the Markov Decision Theory first developed by Benes (1961). We can mention, for instance, the work by Kelly (1988), Dziong et al (1988), Ott and Krishnan (1988). We call the routing models proposed in the precedent references "on-line " since they are real-time models, that is, models that give rules on-line on where to route a call, based on the state of the system. In contraposition to on-line models, there are planning models (off-line) that help the planner in the design, dimensioning or routing policy evaluation of the network. The objective of this paper is to propose an off-line optimization routing model that would integrate several pertinent factors to be considered at the planning stages and that would be versatile and easily solved for real-sized networks. A network reliability application of the model taking into account just one of the factors can be found in Sansó, Soumis and Gendreau (1991). In this paper we show how other factors can be incorporated into the model and we pay particular attention to the computational details of the implementation.

The paper is organized as follows. In section 2, the model and its applications are presented. The resolution approach is discussed in section 3 and results are shown in

section 4. Concluding remarks are exposed in section 5.

2 The model

The classical planning model used in telecommunication networks is the elegant multicommodity formulation such as the one proposed by Girard (1990, chapter 4). According to this formulation, the network is made up by switching stations (nodes) and links (arcs) between the switching stations and there is an origin-destination matrix demand that reflects the traffic needs of the network for a given period of time. The arcs have a capacity in terms of number of circuits. Every origin-destination pair of nodes constitutes a different type of call, so,in mathematical terms, it constitutes a different commodity. The demand will be assigned to the arcs of the network so that some cost or performance function is optimized and the capacity constraints of the network are respected. In general, this type of model is difficult to solve due to the capacity constraints linking different commodities. One way of solving this type of problem is by Lagrangean relaxation. The flaw of this method is that it implies time-consuming repetitive adjustment of the lagrangean multipliers. Sansó (1988) proposed a different model where the capacity constraints are replaced by a penalty function that makes the problem easier to solve and that takes into account the stochastic character of the demand. In the model, the best penalty function is given a priori and no other adjustment is needed during the resolution procedure. A particular application of this model can be found in Sansó, Soumis and Gendreau (1991). In this paper, the model is being extended so that several performance factors are taken into account.

Let $G(N, M)$ represent the graph made up by the switching stations and the links between the switching stations, N is the set of nodes and M the set of arcs. For every arc $u \in M$, there is a capacity in number of circuits C_u, and a total offered traffic X_u. Now, if we define the commodites as being the type of demand from the same origin, then the set of commodities corresponds to the set of origin nodes O. Then, the model proposed, in its most general form can be written as:

$$\min_{X \in \Omega} f(X) \tag{1}$$

In equation (1), Ω represents the set of flows so that flow conservation and non-negativity constraints are respected, X is the vector of the link offered traffic and f is a

function of X. The model implicitly considers the flow conservation of the offered traffic, which is not exactly the case in networks with blocked-calls-lost policies. The accurate model should be, instead, a generalized multicommodity flow problem with a non-linear gain function. Such problems are extremely difficult to solve and, to our knowledge, no algorithm has been yet conceived to efficiently solve these problems. For a discussion on this type of problems, see Girard (1990), chapter 4. We would also point out that the capacity constraints do not appear in the model, they will, instead, be replaced by a penalty term included in the objective function, we leave this particular discussion for later. The objective function of the model re-groups, besides, several interesting planning features. We have specifically considered three features: 1) reduce blocking as much as possible; 2) reduce the number of links necessary to route a call; 3) equilibrate losses in the network. Let f_1, f_2 and f_3 correspond, respectively, to the functions to achieve the three planning goals, then the proposed model can be stated as follows:

$$\min_{X \in \Omega} (k_1 f_1 + k_2 f_2 + k_3 f_3) \tag{2}$$

where k_1, k_2 and k_3 correspond to weights given to each of the planning features of the model. We now present the details of each of the planning features.

Blocking Reduction

This factor was the only one taken into account in the application presented by Sansó, Soumis and Gendreau (1991). One way of indirectly reducing the blocking, was to minimize the lost call traffic. Therefore, Erlang's expected lost call traffic function was used so that f_1 had the following expression:

$$f_1 = \sum_{u \in M} P_u(X) \tag{3}$$

where P_u can be expressed as a function of B, Erlang's loss function:

$$P_u(X) = \sum_{p \in O} X_u^p B(C_u, \sum_{p \in O} X_u^p). \tag{4}$$

We remind the reader that $B(s,a)$ represents the proportion of the traffic to be lost in a link with a capacity of s circuits when the offered traffic is a. The expression for B is given by:

$$B(s,a) = \frac{a^s/s!}{\sum_{i=1}^{s} a^i/i!}. \tag{5}$$

Function f_1 corresponds, then, to the penalty function that "naturally" replaces the capacity contraints of the original multicommodity flow formulation. We used the term "naturally" to emphasize that it is a measure that the planner would like to minimize in designing the routing. Besides, this function, being stochastic, takes into account the stochastic nature of the demand. Thus, by minimizing just function f_1, the planner would be indirectly reducing the blocking.

Reducing path lenght

In on-line routing models, the path lenghts to route an origin-destination call are very often limited to a few arcs. In the model we have presented, there is no notion of pre-specified origin-destination routes, therefore, the path lenghts cannot be limited. However, it has been found (see, for instance, Kelly (1986)), that routing calls in paths of long lenghts can have an adverse effect on the overall blocking performance of the network. One way to take into account this problem in a centralized model such as the one presented is by adding a shortest-path term to the objective function. Function f_2 can thus be expressed as:

$$f_2 = \sum_{u \in M} l_u X_u. \tag{6}$$

In (6) l_u represents the lenght of arc u.

Equilibrating losses in the network

In the model presented, there are no restrictions with respect to the paths a particular origin-destination could use to route its flow. Therefore, since our model optimizes global performance in the network, there is the possibility that in order to achieve such

a performance, some regions will be favored in detriment of other regions. This is not necessarily desirable. We consider, therefore, that it is important for the off-line planning model to have a feature to avoid this situation. To equilibrate losses, a quadratic term of the link expected lost call traffic could be added to the objective function, that is, a term such as:

$$\sum_{u \in M} P_u^2(X). \tag{7}$$

From experience in quadratic programming, one can expect that if function f_3 is set to be equal to expression (7), there is the possibility that the uniformization of the arc losses will be achieved by removing flow from high capacity arcs and blocking more flow on low capacity ones. This situation is not desirable either. Therefore, we propose a function f_3 that would consider the arc capacity:

$$f_3 = \sum_u (P_u(X)/C_u)^2. \tag{8}$$

The precedent expression favors the uniformization of the losses per unit of capacity, which is a much more balanced aim for the planning model.

Applications

In this section we show how to incorporate the routing package into different planning schemes. First of all, in figure 1, the features of the routing package are depicted. The package has two type of entries: a changing demand and the objective function. In what concerns the output, it is possible to obtain the expected number of arcs needed to route a call and flow and overflow variables per origin, per origin-destination and per arc.

On the other hand, in figure 2 the general idea of how to incorporate the routing package into planning models is depicted. The routing model can be used just to evaluate a routing policy under different aspects such as blocking, shortest paths, etc. taking or without taking into account failure scenarios. In the reliability evaluation scheme proposed by Sansó, Soumis and Gendreau (1991), a performance measure was evaluated

for the most probable states of the system, a state being defined as the state of all the network components. Such a scheme, which incorporated the notions of routing and reliability, was later extended to the optimal dimensioning of circuit-switched networks taking into account reliability considerations. A special algorithm for this case can be found in Sansó, Gendreau and Soumis (1991). In this paper, we will not enter into the details of the dimensioning procedure. Rather, the results on the network performance evaluation procedure will be used to show how a routing policy can be assessed under reliability considerations.

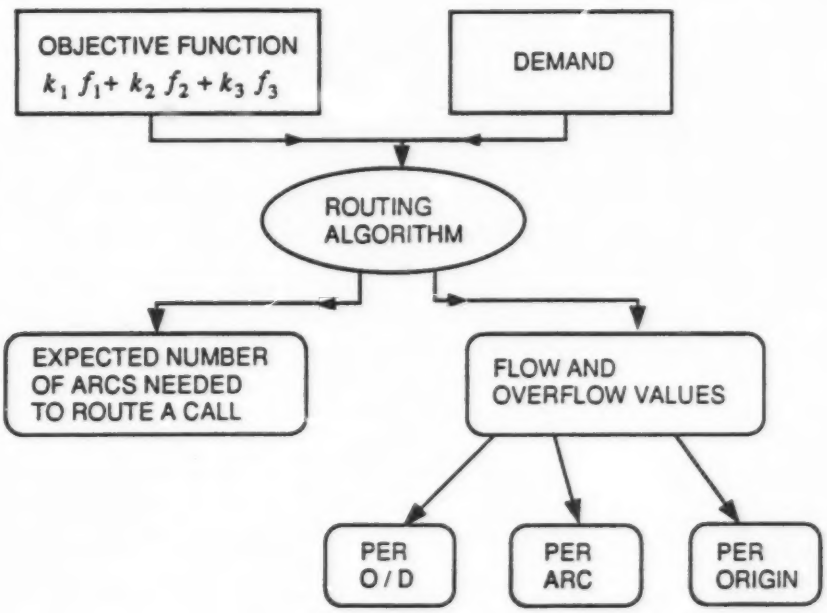

Figure 1: **Features of the routing package**

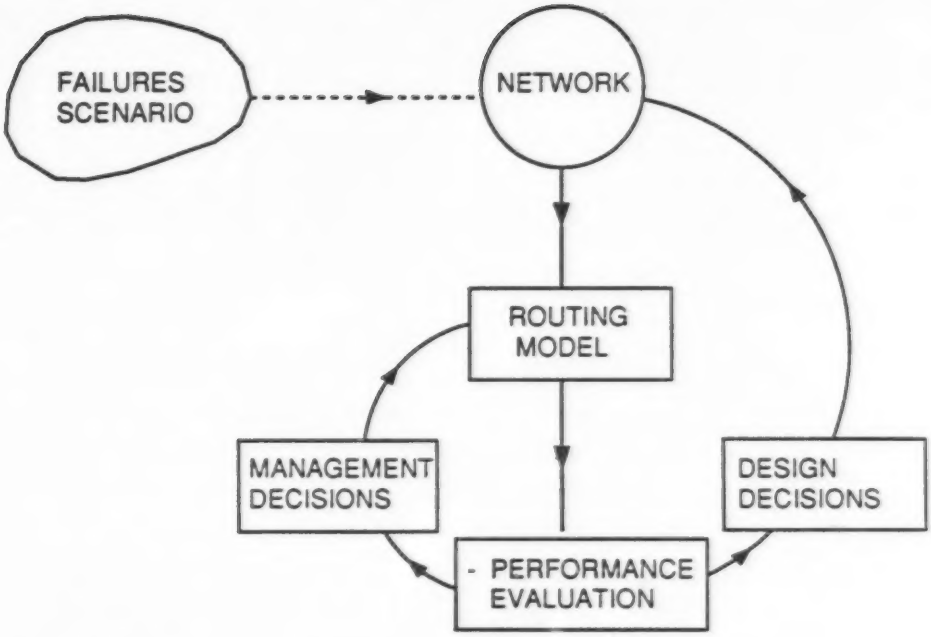

Figure 2: **Routing and planning, how do we incorporate it**

3 Resolution approach

The algorithm used to solve the routing model is a Frank-Wolfe decomposition. The implementation is similar to the one used in transportation planning (see, Guelat (1983)) in which the variables kept at every step of the procedure are the total link flows, whereas the subproblem corresponds to finding the shortest paths for every commodity. This method is known in the telecommunication literature as the "flow-deviation method" (see Fratta, Gerla and Kleinrock (1973)). The advantage of the Frank-Wolfe over other types of resolution approaches is simply its ease of resolution and its availability. However, when much precision is required in finding the objective function, other procedures, such as the Convex Simplex should be considered (see Sansó, Soumis and Gendreau (1991)). The convergence of the algorithm applied to this model is theoretically guaranteed by the fact that the objective function f of the model is convex with respect to the total link flow variables. In fact, the expected lost call traffic function $P_u(X)$ has been proven to be convex over variable X (Palm (1957), a different proof can be found in Sansó (1988)).

We now discuss some technical difficulties that were encountered at the resolution level. First of all, function P_u, which is a function of two variables, the arc capacity and the arc flow, has a complicated form, not easy to incorporate into a procedure chosen to be simple and speedy. Therefore, an approximation was chosen. However, one of the problems encountered was how to evaluate the real value of the function before doing the approximation. In fact, with the advent of fiber optics, a link capacity can be in the tens of thousands of circuits, which implies the evaluation of the factorial of such a capacity. Thus, in order to exactly evaluate the function for particular capacity levels, a computational trick, similar to the one showed by Jagerman (1973) for the computation of Erlang's loss function $B(s,a)$ was used. We refer the reader interested in such details to the appendix.

The approximation

Erlang's expected lost call traffic function, $P_u(X)$ was found to behave as a three piece function: zero when $X_u << C_u$, a nonlinear function when X_u is closed to C_u and $X_u - C_u$ when $X_u >> C_u$. It was also found that as the capacity increases, the nonlinear part of the function decreases. Therefore, for very large capacity values, the function tends toward a two-piece linear function. A good approximation of the function was

found to have the following expression:

$$\hat{P}_u(X) = \begin{cases} 0 & \text{if } X_u \leq C_u - \epsilon, \\ AX_u^2 + BX_u + \xi & \text{if } C_u - \epsilon \leq X_u < C_u + \epsilon, \\ X_u - C_u & \text{if } X_u \geq C_u + \epsilon. \end{cases} \tag{9}$$

In order for $\hat{P}$ and $\hat{P}'$ to be continous A, B and ξ had to have the following values:

$$A = \frac{1}{4\epsilon}; \quad B = \frac{\epsilon - C_u}{2\epsilon}; \quad \xi = \frac{(C_u - \epsilon)^2}{4\epsilon}. \tag{10}$$

The ϵ was chosen so that it is made smaller as C_u increases, therefore the approximation behaves as the real function as C_u increases: it tends toward a two-piece linear function for large values of capacity.

Figures 3 and 4 depict the behavior of the approximation for two different levels of link capacity. In the figures it is shown that for larger levels of capacity the approximation seems to fit less than for small levels. This behavior was designed on purpose. In fact, the asymptotic behavior of the real function for large capacity values would have created problems at the optimization level when arc capacities are large and the total arc flow is closer to the link capacity. An algorithm such as the Frank-Wolfe would spend too many iterations in finding the optimal solution for such conditions. An upper bound on the maximum error made by the approximation was found to be $\epsilon/4$, which, in practical terms could be translated into 4% of the capacity. However, for the data set obtained, the maximum error found was in the order of 0.3% of the capacity. Moreover, the error is always made in the same sense, the approximation is thus an upper bound of the real value.

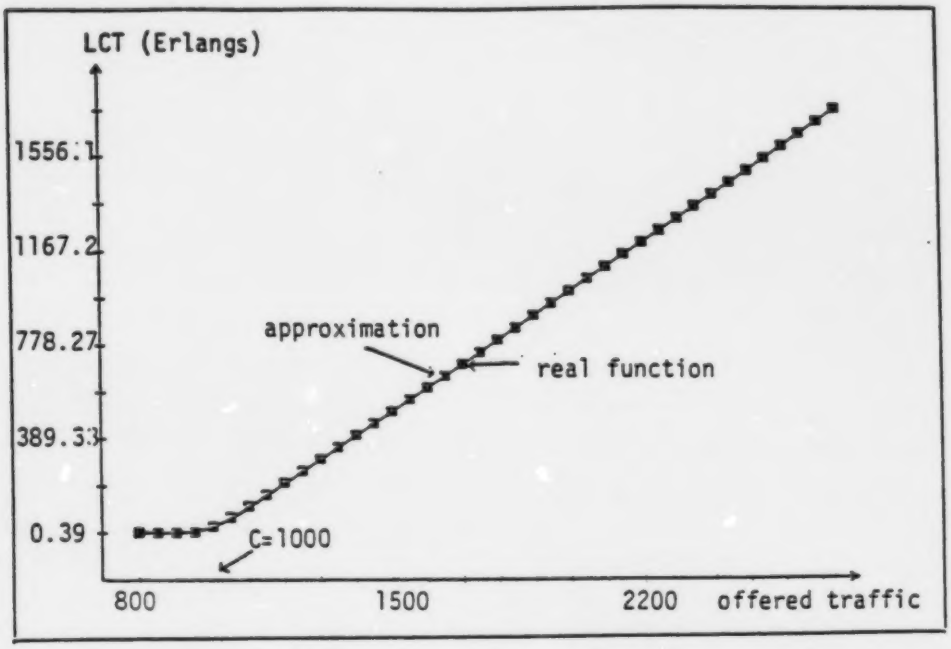

Figure 3: **Approximation of $P_u(X)$ for $C = 1000$**

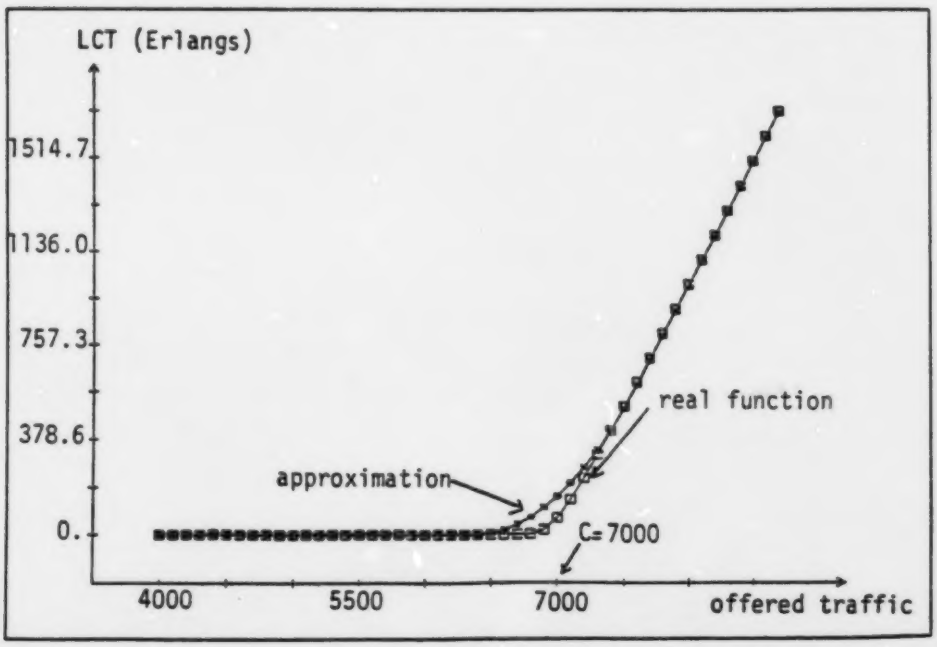

Figure 4: **Approximation of $P_u(X)$ for $C = 7000$**

4 Results

The network

The network was made up from real life specifications obtained from a telephone company. In figure 5 the network topology has been depicted. The nodes are represented by squares and the node numbers are presented at each square corner. Arc capacities, in thousand of circuits, are depicted in brackets on every arc. The vertical dashes on the arcs represent the units of arc lenght. The demand matrix used is available from the authors.

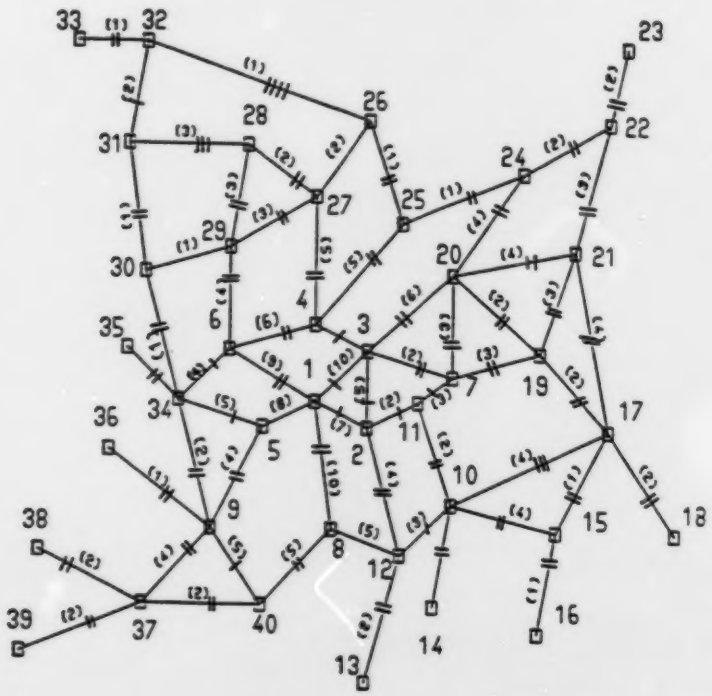

Figure 5: RED network

Behavior of the routing algorithm

To study the behavior of the algorithm, k_1 was set to 1 and k_2 and k_3 were set to 0. The level of demand and the CPU time, on a SUN 3/50, was computed. The computation was done for two variants of the algorithm. The first one considered the commodities to be the origin nodes, this is the model described in section 2. The second variant is the more classical one in which the commodities are the origin-destination pairs. This second version of the problem is much more time consuming. In figure 6, it is shown how the behavior of the algorithm is exactly the same in the two cases.

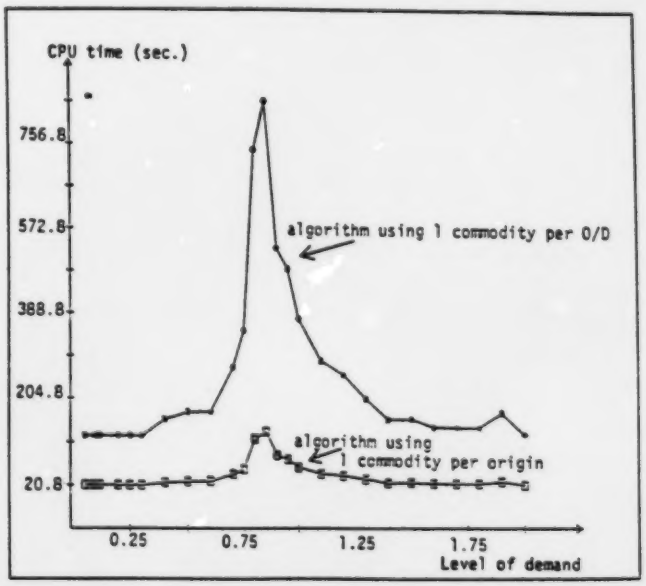

Figure 6: **Behavior of the routing algorithm**

As it can be appreciated from the figure, for small and high levels of demand the CPU time is, in relative terms, short. When the demand is approaching 80% the basic demand, there is an increase in CPU time. This behavior is explained by the shape of the objective function and by the characteristics of the Frank-Wolfe algorithm. The objective function is separable by arc and on each arc, the function considered is the approximation $\hat{P}_u(X)$ which is linear at the extremes (low and high offered traffic) and quadratic for offered traffic values closed to the capacity. Since the Frank-Wolfe algorithm solves a

linear subproblem, if the objective function were linear, the optimum solution would be found in one complete iteration. Therefore, when the demand levels are such that the objective function is linear for most of the arcs, the algorithm finds the solution in few iterations. This is the case for very low and very high levels of demand. On the other hand, when the demand levels are so that the objective is non-linear for most of the arcs, the algorithm takes longer time to reach the optimum. As it can be appreciated from the figure, there is even a demand value for which the algorithm takes a maximum number of iterations to solve the problem.

Path lenght reduction

In order to study the behavior of the flow with respect to the introduction of the shortest path term, the mean number of arcs needed to route a call (MNARC) was studied for different levels of demand. In figure 7 the MNARC variation with respect to the weight of the shortest path term was shown. For this test, k_1 was set to 1, k_2 varies and k_3 was set to zero.

The figure shows that the addition of a shortest path term to the objective function does have a definite impact on the MNARC. The most dramatic decrease is obtained when the level of demand is 80% the basic demand, on the other hand, the lowest benefic impact is gotten for the 200% curve. This, again, can be explained by the shape of the objective function for the different levels of demand. In fact, it has been seen, already, that when the demand matrix approaches 80% the basic demand, the objective function presents a definite non-linear character, hence the introduction of an SPT will have much influence on the outcome of the routing. Conversely, for very high levels of demand, the objective function presents an almost totally linear behavior, the flow will "naturally" be assigned to the shortest path and, therefore, the addition of a SPT to the objective function will have a mild effect on the overall routing. Another interesting result that should be pointed out is the asymptotic behavior of the MNARC over the weight given to the SPT. This indicates that there is a lower bound on the MNARC which is inherent to the topology of the graph and independent of the level of demand.

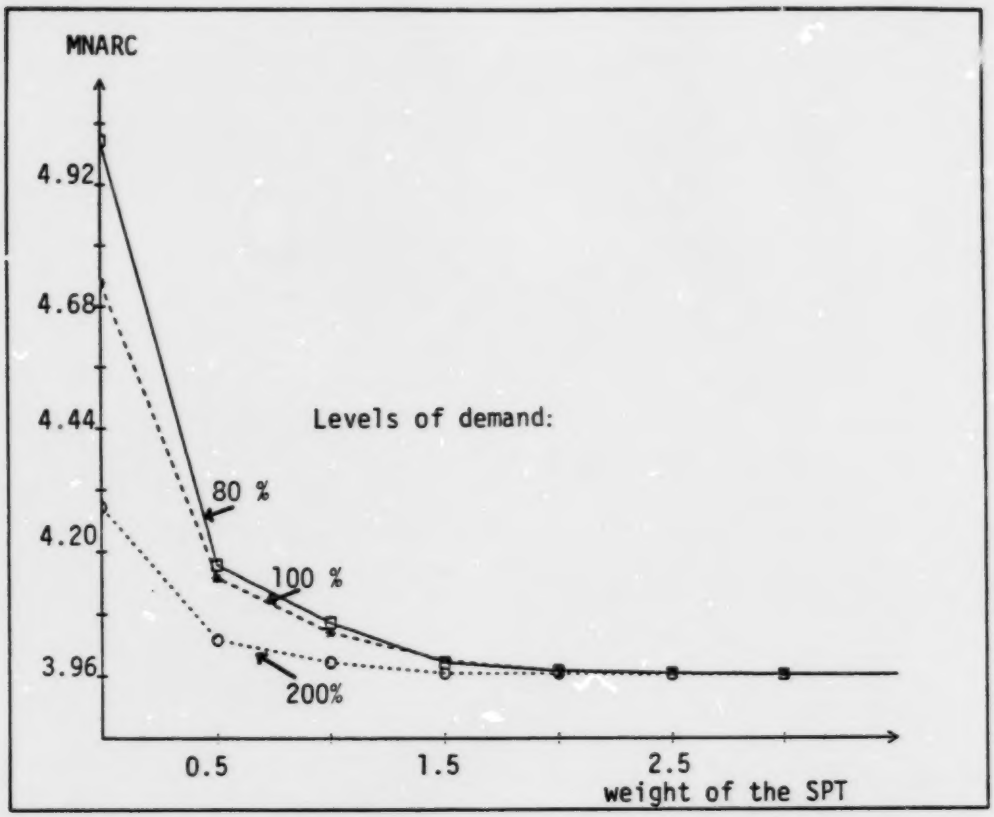

Figure 7: **Variation of MNARC with respect to k_2**

Equilibrating losses

As pointed out in the previous section, a special term, f_3, was added to the objective function in order to equilibrate the percentage of losses per unit of capacity in the network. In figure 8 the variance on the percentage of losses per unit of capacity is depicted with respect to k_3, the weight of the third term of the objective function. For such a figure, k_1 was set to one and k_2 to zero. The percentage of losses per capacity unit per arc was evaluated as $[P_u(X)/C_u] * 100$ and the variance was taken over all arcs of the network. As expected, the increase of k_3 implies a decrease of such a measure.

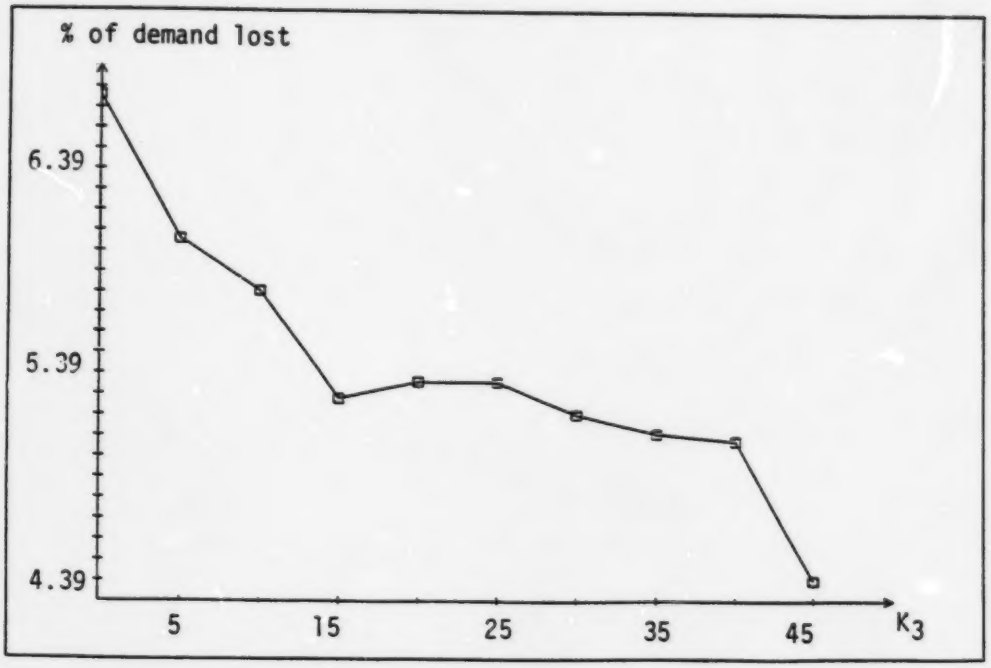

Figure 8: **Variance of the percentage of losses vs k_3**

Network performance evaluation

In this section we consider some features of the routing model that could be applied to evaluate a measure of network performance given that links can fail independently with a given probability. Lets suppose that only link can fail and that there is a probability of failure associated with every link. Then, a good measure of network performance was considered the expected lost call traffic (see Sansó, Soumis and Gendreau (1991)). In their paper, they described how to calculate an upper bound on the performance measure, the procedure can be resumed as follows: 1) enumerate the most probable states of the system, 2) for each of those states solve an optimal routing problem and evaluate the lost call traffic, 3) the upper bound of the expected lost call traffic is made up of the constribution to the measure of the most probable states, for all other states it is considered that the traffic lost corresponds to the worst possible case. We refer the reader interested in details to the article previously mentioned.

In this paper we have presented just a result on the impact of routing in the measure of performance. When a link failure takes place, is it beneficial or not to re-route the flow otherwise lost by the failure? We have tried to answer those questions by evaluating network performance under two different routing policies: lose the flow on the failed link or re-route the flow when the link fails. For this, three different cases were studied: case 1 corresponded to an uncongested network, case 2 was a network with 80% the standard demand, case 3 was a pretty congested network (at 100% the standard demand) and case 4 represented an extremely congested network (150% the standard demand). The two routing policies were evaluated for different levels of failure probability. In table 1 Mdty represents the mean down time per year (in hours) of a unit of length in the network, this measure is used to compute the network probability of failure. For everyone of the cases considered, the upper bound on the expected lost call traffic under the two different routing policies were evaluated. The table presents the percentage of improvement of the re-routing policy over the loss policy. In other words, if Z_r represents the upper bound on the expected lost call traffic with re-routing after failure and Z_l represents the upper bound on the expected lost call traffic without re-routing, then the % of improvement depicted in the table is calculated as $((Z_l - Z_r)/Z_r) * 100$.

From the table, it can be observed that the beneficial effect of re-routing after a failure decreases with congestion. This is probably due to the fact that when the network is very congested, re-routing the flow could cause overloading in adjacent links and could be detrimental. For case 4, the most congested network considered, even a negative percentage of improvement from re-routing was found. On the other hand, if there is a low blocking level in the network, re-routing the calls is definite beneficial. Now, it can be seen that, for a given case, that is, for a fixed demand matrix, the beneficial or detrimental effect of re-routing increases with the probability of failure in the network.

To conlcude this section, results for a low blocking network example has been displayed in figure 9. This figure depicts the upper bounds on the expected lost call traffic for the two routing-after-failure policies versus the Mdty. From the figure, it can be seen that the expected lost call traffic grows more than linearly with the increase in the probability of failure. Also, it can be pointed out that the two curves grow farther apart as the failure probability increases. In other words, as it was mentioned earlier, the (in this case) beneficial effect of re-routing after failures increases with the probability of failure.

Mdty	%Improvement			
	case1	*case2*	*case3*	*case4*
0	0	0	0	0
30	1.13202	0.04790	0.0055	-0.00022
50	1.87700	0.07973	0.00554	-0.00037
70	2.62343	0.11145	0.01663	-0.00051
90	3.36304	0.14259	0.01663	-0.00066
120	4.81716	0.18934	0.02217	-0.00089
140	5.18288	0.22021	0.02771	-0.00104
160	5.89681	0.25096	0.02770	-0.00118
180	6.60351	0.28111	0.03323	-0.00133
200	7.29837	0.31161	0.03877	-0.00147
140	8.65348	0.37174	0.04428	-0.00177
300	10.5968	0.46020	0.05531	-0.00220
160	12.4173	0.54638	0.07185	-0.00265
400	13.5525	0.60275	0.07734	-0.00293
500	16.1130	0.73980	0.09377	-0.00363
600	18.2558	0.86983	0.11015	-0.00432
1000	83.2148	1.32626	0.18043	-0.00701
1500	23.5294	1.75018	0.25947	-0.01017

Table 1: **% of improvement of the re-routing vs loss policy**

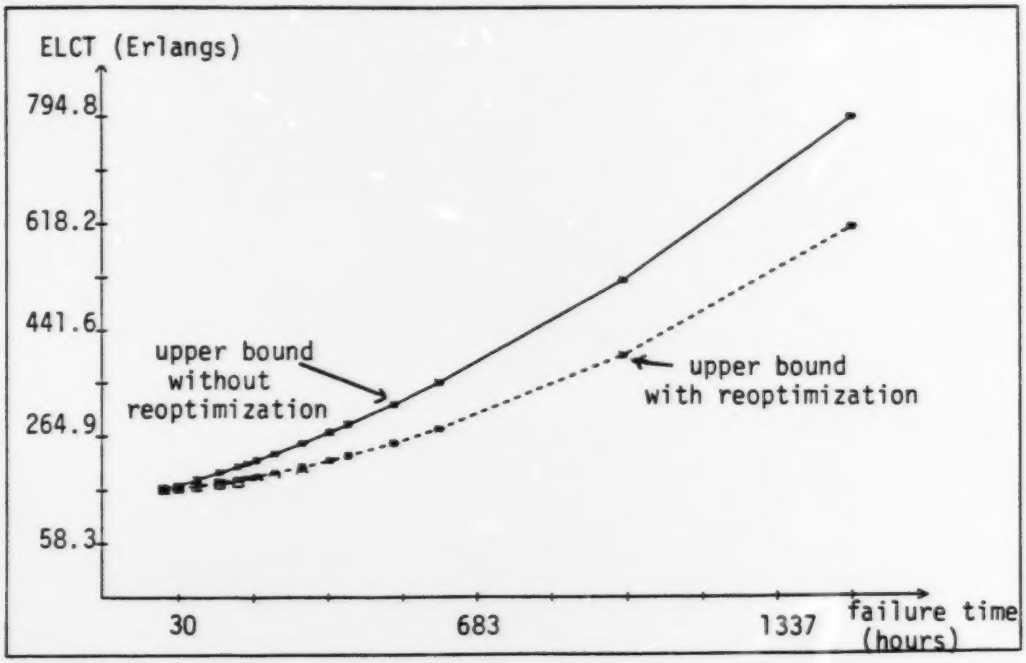

Figure 9: **Comparison of bounds with and without re-routing option after a failure**

5 Conlusions and further work

In this paper, an off-line routing model taking into account several planning factors for circuit-switched networks has been presented. The formulation and the computatinal features of the model have been throroughly discussed to show how all different numerical and modeling pieces can stand together to efficiently solve real-size problems.

Several applications of the model has been surveyed and results have been compiled. For some of the applications, real time routing features such as blocking reduction, limiting the number of path links and equilibrating losses in the network, were reproduced. It was shown how the model provided a good insight into the routing possibilities of a given network without the need of solving time-consuming on-line routing simulations.

The model can also be used for network performance evaluation under different routing policies. In fact, it could be helpful in determining when/if a particular option, such as re-routing flow after a failure, would be desirable or not in a stochastic network under specific traffic conditions. In our empirical examples, it was found that the improvement caused by re-routing decreases with congestion, and when the network is very congested, re-routing after a failure could be detrimental. It was also found that, whatever the level of blocking, the improvement or the detrimental effect of the re-routing option increases with the probability of failure of the network.

The routing model has also been extended to network design and dimensioning taking into account reliability considerations. We refer the interested reader to Sansó, Gendreau and Soumis (1991). We are currently working on decentralized off-line planning models and on the extensions of such model to packet-switched networks.

Appendix

We have kept for the appendix the computational trick that allows the exact evaluation of $P_u(X)$ for large values of C. We point out that such evaluation was made just once, to be able to find a suitable approximation of the real function. Let $F_u(X)$ represent the inverse of function $P_u(X)$ such that:

$$F_u(X) = \frac{1}{P_u(X)}, \tag{11}$$

which can also be written as:

$$F_u(X) = \sum_{t=0}^{C_u} \frac{C_u!}{C_u - t} \frac{1}{X^{t+1}}. \tag{12}$$

The precendent expression can be reformulated as:

$$F_u(X) = \sum_{j=0}^{C_u} A_j, \tag{13}$$

where A_j corresponds to:

$$A_j = \begin{cases} 1/X & \text{if } j = 0 \\ \frac{C_u-(j-1)}{X} A_j & \text{if } j = 1, ...C_u. \end{cases} \tag{14}$$

Expression (14) allows the evaluation of $P_u(X)$ by means of a simple do loop performing a product C_u times.

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